Generalized Sharpe Ratio:
A Defense Against Sharpened Sharpe Ratios

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Abstract

Recently, a number of authors have shown that by using dynamic trading strategies and/or investing in options, a portfolio's Sharpe ratio can be increased even when the investor has no 'skill'. The increased Sharpe ratio comes about by creating a return distribution that is significantly different from normal. This paper presents a generalized approach to calculating the Sharpe ratio of an asset or a portfolio with a return distribution that is not necessarily normal. The procedure adjusts the entire distribution of the asset's return so that it will match the return distribution of a benchmark (e.g., S&P500). The Sharpe ratio of adjusted return can then be directly compared to that of the benchmark. We apply the procedure to simulated data and the historical data on hedge fund indices and hedge fund managers.

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1. Introduction

This paper presents a new approach for calculating the Sharpe of an asset or a portfolio.[1] The methodology, first, transforms the return on the asset so that its distribution matches the distribution of a benchmark. Then, it calculates the Sharpe ratio of the asset return using the adjusted distribution.[2] As a result, the Adjusted Sharpe ratio will be directly comparable to that of the benchmark.

Recently, a number of papers have shown that the Sharpe ratio of an investment can be manipulated so that the strategy will have a higher Sharpe ratio than that of a benchmark (see Spurgin (2001) and Goetzmann et. al. (2002)). Even when an investor is not intentionally manipulating the Sharpe ratio of a portfolio, non-linear payoffs and deviations from normality can significantly reduce the applicability of the Sharpe ratio (e.g., see Bernardo and Ledoit (2000)). For example, a covered call strategy will produce a return distribution with low standard deviation and significant negative skewness. Even if the option is sold at its fair market value, the covered call strategy will produce a Sharpe ratio that will exceed the Sharpe ratio of the underlying asset.

Managers of actively managed portfolios may use options or option replicating strategies to produce return distributions that are significantly different from the return distributions of long positions in major equity and bond indices (hedge funds are primary examples of such portfolios). A number of authors have argued that because returns for some hedge fund strategies may not be normally distributed, they cannot be analyzed

[1] For the original discussion of the Sharpe ratio see Sharpe (1966). For a more recent discussion by the original author see Sharpe (1994). For measures of performance that take deviations from normality explicitly into account see Leland (1999), Stutzer (2000), Madan and McPail (2000), and Keating and Shadwick (2002).

[2] For a discussion of the procedures for transforming a random variable so that its histogram matches the histogram of another random variable see Gonzalez and Woods (1992), Johnson et. al. (1994) and Wichura (2001).

using the mean-variance framework and by extension the Sharpe ratio should not be used to measure their performance.[3]

The procedure employed here allows us to transform the distribution of returns from an asset or a portfolio so that it will match the return distribution of a benchmark return. The Sharpe ratio of the transformed payoff can then be compared to that of the benchmark. Similar to the original Sharpe ratio, the Adjusted Sharpe ratio measures each asset or portfolio's performance on a stand alone basis since the correlation of its return with return on other assets is not taken into account (see Sharpe (1992)). When Adjusted or original Sharpe ratio is employed to measure the performance of an investment, the implicit assumption is that the only relevant factors in ranking investments are the mean and the variance of the return distributions.[4]

2. Methodology

Let R denote the random rate of return on a risky asset or a portfolio (e.g., monthly return on a hedge fund portfolio). Also, there exists a benchmark and its rate of return is denoted by B over the same time period. Finally, there is a riskless asset and its rate of return is given by r_f. The classical Sharpe ratio of the benchmark, SR_B, is given by

$$SR_B = \frac{E\left[B\right] - r_f}{Std\left[B\right]} . \qquad (1)$$

It is well known that the above ratio is the appropriate measure of performance of a portfolio if the distribution of its rate of return is normally distributed. While this

[3] See Chris and Kat (2001). For a different view on this see Fung and Hsieh (1999) and Schneeweis et. al. (2002).
[4] The foundation for this type of analysis based on the distribution of the end period payoffs was provided by the Dybvig (1988a and 1988b). The present paper follows the work of Robinson (1998) and Amin and Kat (2001).

assumption is approximately satisfied by monthly returns on broad equity indices, it is usually not satisfied by actively managed portfolios that use dynamic trading strategies or options to alter the payoff distribution of underlying assets. Our approach is to transform the payoff on such an investment so that its distribution will match that of the benchmark. Once the return is transformed, the resulting Sharpe ratio of the asset can be directly compared to that of the benchmark knowing the total payoffs from both investments have exactly the same distribution.

The methodology is as follows: We seek a function $F(x)$ such that

$$F(1+R) \sim 1+B. \tag{2}$$

That is, the distribution of the transformed total payoff from the asset matches the distribution of the payoff from the benchmark. One can think of $F(1+R)$ as the payoff from a derivative security whose future payoff depends on the future value an asset that has an initial price of \$1 and an end of period value of $1+R$. We know that the initial investment required to generate $1+B$ through an investment in the benchmark is \$1. Assuming that the function $F(x)$ is estimated, we are able to create the same payoff distribution through an investment in the "derivative" security with the payoff function of $F(1+R)$. However, we need to determine the initial value of this derivative security. We use the risk-neutral option pricing approach to price the initial value of $F(1+R)$. Using this approach, the current value of $F(1+R)$ is given by:

$$P = \frac{E^Q\left[F(1+R)\right]}{1+r_f}, \tag{3}$$

where $E^Q[\ \ ]$ means expectation under the risk neutral probability distribution.

Under this approach, the expected value of the payoff is calculated under a probability distribution that would prevail if investors were completely risk neutral. Once the expected value of the payoff under this probability distribution is estimated, then its present value is calculated by discounting it at the risk-free rate. Equation (3) calculates the current price of a "derivative" security that is designed so that its payoff has exactly the same probability distribution as that of the benchmark.

The required initial investment in the benchmark that will generate $1+B$ is \$1. The "derivative" security with its price determined in equation (3) generates the payoff given by $F(1+R)$, which has exactly the same distribution as $1+B$. Since the initial price of the derivative is given by P, the expected return on the derivative security is given by

$$\frac{E[F(1+R)]}{P}-1=\frac{E\left[B+1\right]}{P}-1. \tag{4}$$

Clearly, if $P<1$, the derivative security is a better investment than the benchmark for the distribution of its payoff matches the benchmark's and yet it costs less than \$1 to generate same payoff distribution. Since we can use the actively managed portfolio to create the same payoff as the benchmark's at a lower cost, we can conclude that the actively managed portfolio outperforms the benchmark. By the same token, if $P>1$, then the benchmark outperforms the actively managed portfolio.

Given the estimated value of the derivative security, we can relate the Sharpe ratio of the transformed return to the Sharpe ratio of the benchmark. In particular, noting that $E\left[F(1+R)\right]=E\left[B\right]+1$ and $Std\left[F(1+R)\right]=Std\left[1+B\right]=Std\left[B\right]$, the Adjusted Sharpe ratio (ASR) of the investment is:

$$ASR = \frac{\frac{E\left[F(1+R)\right]-P}{P} - r_f}{Std\left[\frac{F(1+R)-P}{P}\right]} = \frac{\frac{E\left[1+B\right]-P}{P} - r_f}{\frac{Std\left[1+B-P\right]}{P}} \tag{5}$$

$$ASR = \frac{E\left[B\right]-r}{Std\left[B\right]} + \frac{(1+r_f)(1-P)}{Std\left[B\right]}$$

We can see from the last line of above equation that the difference between the Sharpe ratio of the benchmark and the Adjusted Sharpe ratio of the portfolio is given by $(1+r_f)(1-P)\big/Std\left[B\right]$. If $P<1$, this term will be positive, and the actively managed portfolio outperforms the benchmark.

In some instances it will be desirable to reverse the above process by transforming the benchmark's payoff so that its distribution will match the distribution of the actively managed portfolio's payoff. In this case the transformation represents a derivative security that its payoff is contingent on the benchmark's return and is designed such that its payoff distribution matches the payoff distribution of the actively managed portfolio. Once the fair price of this derivative security is estimated, we can calculate the Adjusted Sharpe ratio of the benchmark, which can be directly compared to the Sharpe ratio of the actively managed portfolio. However, if the distribution of the portfolio return is far from normal, the resulting Sharpe ratio may not be comparable to those of other assets. In the empirical section of the paper we use both approaches to show that one obtains consistent results with either method.

3. Algorithm

In this section we describe an algorithm than can be used to estimate the function $F(1+R)$ and then to calculate P. The algorithm is simple and can be implemented using a spread-sheet program.[5] Suppose we have collected a sample size N of monthly rates of return on an asset and a benchmark. The following steps should be taken:

1. Sort each sample in a descending order. Each observation of the sorted samples of returns will now represent the same point on the empirical distributions of the two investments. For example, the i^{th} observation in each sorted sample refers to the point on the return distribution where $\Pr\left[\text{Ret} \le i^{\text{th}}\text{Ret}\right] = \frac{i-1}{N-1}$. That is, according to the empirical distributions of the returns, the probability that an observed return is less than the i^{th} sorted return is the same for both distributions and equal to $\frac{i-1}{N-1}$.

2. Using the sorted samples, the *total* return on the benchmark is regressed against a function of the *total* return on the asset. In the examples below we use a polynomial to approximate the function $F(1+R)$.[6] That is, the following regression is run to estimate the parameters $\alpha_0, \alpha_1, ..., \alpha_K$:

$$1 + B = \alpha_0 + \sum_{j=1}^{K} \alpha_j (1+R)^j + \varepsilon . \tag{6}$$

   The right hand side of equation(6) is an approximation of function $F(1+R)$.

3. In this step we calculate the fair price of the derivative security that its payoff is given by $\alpha_0 + \sum_{j=1}^{K} \alpha_j (1+R)^j$. We use a bootstrap method to calculate the expected value of $F(1+R)$ under the risk neutral probability. For this reason we

[5] A copy of the program can be requested from the authors.

[6] We also experimented with other forms of approximating the function $F(1+R)$ such has piecewise fitting of polynomials, Hermite polynomials and Gram-Charlier Series (see Johnson et. al. (1994) and Wichura (2001)). We obtained virtually the same results.

need to adjust the observed values of R such that our sample mean is equal to the riskless rate. For this purpose we add $r_f - E\left[R\right]$ to each of the historical values of the portfolio return, where $E[R]$ is the mean of the sample. This forces the mean return of the sample to be the risk-free rate. We then use the adjusted observed values to calculate the expected value of $F(1+R)$ under the risk neutral probability distribution. Let $\hat{R}_j$ denote the j^{th} adjusted return on the asset. The estimated value of P is given by:

$$P = \frac{\frac{1}{N}\sum_{j=1}^{N} F(1+\hat{R}_j)}{1+r_f},$$

where N is the sample size and $F(1+\hat{R}_j) \approx \alpha_0 + \sum_{i=1}^{K} \alpha_i \left(1+\hat{R}_j\right)^i$.

4. Applications

In this section we present several applications of our methodology to show the power and the accuracy of this technique. They include:

1. Simulated returns on a benchmark and a portfolio that consists of a long position in the benchmark and a short position in an out of the money call option. This case was discussed in Goetzmann et. al. (2002). They show that the Sharpe ratio of the portfolio would exceed the Sharpe ratio of the benchmark by about 0.11 even though the option is sold at its fair value. Our Adjusted Sharpe ratio shows that the two investments have the same performance.
2. Monthly returns on HFR and CISDM hedge fund indices, S&P500 index and Lehman Aggregate Bond Index. Recent research has suggested that that returns from some hedge fund strategies may not be normally distributed. Therefore,

traditional Sharpe ratio cannot be employed to compare their performances to that of a benchmark such as S&P500 Index or Lehman Aggregate Bond Index. Our results show that most of these indices have indeed out-performed these benchmarks and that in most instances the traditional Sharpe tends to *underestimate* the degree of out-performance of these indices.

3. Monthly returns on 30 hedge fund managers, S&P500 Index and Lehman Aggregate Bond Index. This is similar to the previous example, except that we use data on managers to perform our tests.

4.1 Simulated Returns

In this section we simulate monthly returns on a benchmark that has annualized mean and standard deviation of 15%. The riskless rate is assumed to be 5% per year. We create a portfolio by investing $1 in the benchmark and selling 0.843 units of a call option with the strike price of 1.0098. These are the same figures used by Geotzmann et. al. (2002) and they show that the Sharpe ratio of the portfolio will be 0.748 while the Sharpe ratio of the benchmark is 0.631.

We simulated 30,000 paths of the returns. The following is the graph of the distribution function of the total return on the benchmark and the portfolio. The figure also displays the distribution function of the transformed return (i.e., the payoff from derivative written on benchmark). The procedure for obtaining the transformed distribution is described below.

Figure 1

Basic statistics of the two returns are displayed in Table 1.

Table 1

The figures reported in Table 1 and the graphs displayed in Figure 1 show how the covered call strategy is able to produce a higher Sharpe ratio. The payoff from the strategy is highly skewed and leptokurtotic. Since, the distribution of the return from the covered call strategy is not normal, the classical Sharpe ratio may not be applicable to this investment.

To apply our algorithm, we generated two samples of 120 observations, one for the benchmark and one for the covered call portfolio. Each sample is sorted in a descending order and then the following regression is run using these sorted samples:

$$1+B=\alpha_0+\sum_{i=1}^{4}\alpha_i\left(1+R\right)^i+\varepsilon \tag{7}$$

No surprisingly, the $R-\text{sqaured}$ of the above regression is very high (0.998), and the resulting distribution function of the payoff from the transformed portfolio return is virtually indistinguishable from the distribution function of the benchmark's payoff (see Figure 1). The last row of Table 1 displays the statistics for the distribution of the transformed returns. We can see that the first 4 moments of the distribution of the transformed returns are virtually identical to those of the benchmark.

Since we use simulated returns, we can randomly generate returns on the portfolio. However, we adjusted the mean of the return distribution so that it represents the risk neutral probability distribution of the portfolio. We generated 10,000 random returns from the risk neutral distribution to estimate the value of P using the following expression:

$$P = \frac{\frac{1}{N}\sum_{j=1}^{N}\left\{\hat{\alpha}_0 + \sum_{i=1}^{4}\hat{\alpha}_i\left(1+\hat{R}_j\right)^i\right\}}{1+r_f}, \tag{8}$$

where $N = 10,000$ and $\hat{\alpha}_i$ are the estimated coefficients of the regression displayed in equation (7). The estimated value of P is 1.0003, which as expected, is not significantly different 1, and thus, the Adjusted Sharpe Ratio of the portfolio (0.67) is not significantly different from the Sharpe Ratio of the benchmark (0.68; also see last column of Table 1).

Though this example is interesting from a conceptual framework, it is not the ideal example for the application of our algorithm. The reason is that the portfolio consists of the benchmark and an option on the benchmark. Thus, the procedure described above basically reverses the effects of the option on the portfolio's return, and thus quite successfully replicates the distribution of the benchmark's payoff using the payoff from the portfolio.

4.2 Hedge Fund Indices

In this section we use a subset of CISDM (formerly known as MAR) and HFR hedge fund indices to apply our methodology. The following table presents summary statistics of these hedge fund indices, S&P500 index and Lehman Aggregate Bond index for the period of January 1990 through September 2002.

Table 2

Hedge fund managers are known for their flexibility to follow various dynamic trading strategies. Given the estimates reported in Table 2, we can see that returns on some of these indices may not be normally distributed, and thus, they are perfect subjects for our experiment. The last column of the table presents the benchmarks that we are going to use to evaluate the performance of each hedge fund index. We can see from the above table that except for CISDM and HFR Emerging Markets indices and HFR Fixed Income Arbitrage index, all other indices have Sharpe ratios that are higher than the Sharpe ratios of S&P500 Index and Lehman Aggregate Bond Index.

For illustration purposes Figures 2 and 3 display the distribution functions of HFR Convertible Arbitrage index, HFR Equity Market Neutral index, S&P500 index and Lehman Aggregate Bond index as well as the transformed return from the two hedge fund indices.

Figures 2 and 3

We can see that the raw distributions of these hedge fund indices are very different from the distribution of their corresponding benchmarks. Further, we can see that the transformed returns can replicate the return distributions of their benchmarks rather well. The graphs of the raw returns and the summary statistics clearly indicate that the distribution of returns from hedge fund indices and the two benchmarks are very different, and further the return distributions of the hedge fund indices do not appear to be normally distributed.

We performed our algorithm for the hedge fund indices. To approximate the function $F(1+R)$, we fitted a polynomial to the sorted returns on the indices. The results are presented below:

Table 3a

Table 3a displays the results for those hedge fund indices that Lehman Aggregate Bond Index serves as the benchmark. The first row displays various monthly statistics for the benchmark. The next row displays the same pieces of information for the HFR Convertible Arbitrage Index plus three additional performance measures. The first is the estimated value of P when the payoff on HFR Convertible Arbitrage Index is transformed to replicate the payoff of Lehman Aggregate Bond Index. We can see that it costs slightly less than $1 to produce the same payoff using HFR Convertible Arbitrage Index. The last column represents the estimated value of P when the above process is reversed. That is, the payoff from Lehman Aggregate Bond Index is transformed to replicate the payoff from HFR Convertible Arbitrage Index. As expected, it costs more than $1 to use Lehman Aggregate Bond Index to replicate the payoff from HFR Convertible Arbitrage Index. The properties of the transformed payoffs are presented in the next row. We can see that for HFR Convertible Arbitrage Index the transformed return matches the first four moments of the distribution Lehman Aggregate Bond Index with a high degree of precision. The remaining rows of Table 3a follow the same pattern.

Three broad conclusions can be drawn from the results displayed in Table 3a. First, our procedure is robust enough such that return on various hedge fund indices can be used to replicate the payoff of the same benchmark.[7] By comparing the properties of the transformed payoffs and the benchmark return, we can see that the transformed returns have similar distributional properties as the benchmark. Second, the hedge fund indices considered here have, by various degrees, outperformed the Lehman Aggregate

[7] The R-squared of the all regressions exceeded 0.98 with most exceeding 0.99.

Bond Index during the past 12 years. Third, the Adjusted Sharpe ratios are not significantly different from the traditional Sharpe ratios. This means that though the return distributions of hedge indices are different from that of Lehman Aggregate Bond Index, the differences are not significant enough to change the Sharpe ratio drastically.

The table below presents similar results for those hedge funds indices that S&P500 Index is used as the benchmark.

Table 3b

Again, we can see that all of these hedge fund indices have outperformed the S&P500 Index during the last 12 years. However, in two cases the traditional Sharpe ratios have to be adjusted down if they are to be compared to the Sharpe ratio of the benchmark.[8]

4.3 Hedge Fund Managers

In this section we use monthly returns on a group of 30 hedge fund managers that report to CISDM hedge fund database. Table 4 presents summary statistics of these hedge fund managers, S&P500 Index and Lehman Aggregate Bond Index for the period of January 1995 through September 2002.

Table 4

We followed the algorithm that was discussed in Section 3. The results appear in Tables 5a and 5b.

Table 5a

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[8] The R-squared of the regressions performed here were never less than 0.97, and from the properties of the transformed returns we can see their distributions closely match the distribution of S&P500 returns

We can see that Distress Securities Manger 3 is the only one with a P value that is greater than 1. The Adjusted Sharpe ratio of this manager is reduced from 0.22 to 0.12. In the case of Convertible Arbitrage Manager 5, the Adjusted Sharpe ratio is lower than the traditional Sharpe ratio though its estimated value of P is less than 1. Overall we can see that returns for these hedge managers can be transformed to replicate the payoff from the benchmark. None of the $R-\text{sqaured}$ of the regressions were less than 0.97 and we can see from the properties of the transformed returns that they match the distribution of the Lehman Aggregate Bond Index rather well.

Table 5b

Table 5b represents the results for those hedge managers for whom the S&P500 Index is used as the benchmark. We can see that in one case the estimated value of P is greater than 1 and that in two cases the adjusted Sharpe ratios are lower than the original Sharpe ratios.

7. Summary

This paper presents a new methodology for adjusting the Sharpe ratio of portfolio returns when their returns are not normally distributed. The procedure transforms the distribution of the investment so that it will match the distribution of the benchmark. After taking the cost of this transformation into account, we can use the transformed returns to compare the Sharpe ratio of the investment to that of the benchmark.

We applied the methodology to a sample of hedge fund indices and hedge fund managers. The results showed that in many instances the Adjusted Sharpe ratio is not significantly different from the original Sharpe ratio indicating that the deviations from normality were not significant enough to affect the Sharpe ratios. We also applied our methodology to simulated data using an example provided by Goetzmann et. al. (2002). The results showed that the methodology can correctly measure the cost of deviation from normality and that the Adjusted Sharpe ratio of a covered call position is equal to the Sharpe ratio of a long position in the underlying asset.

6. Figures

Figure 1

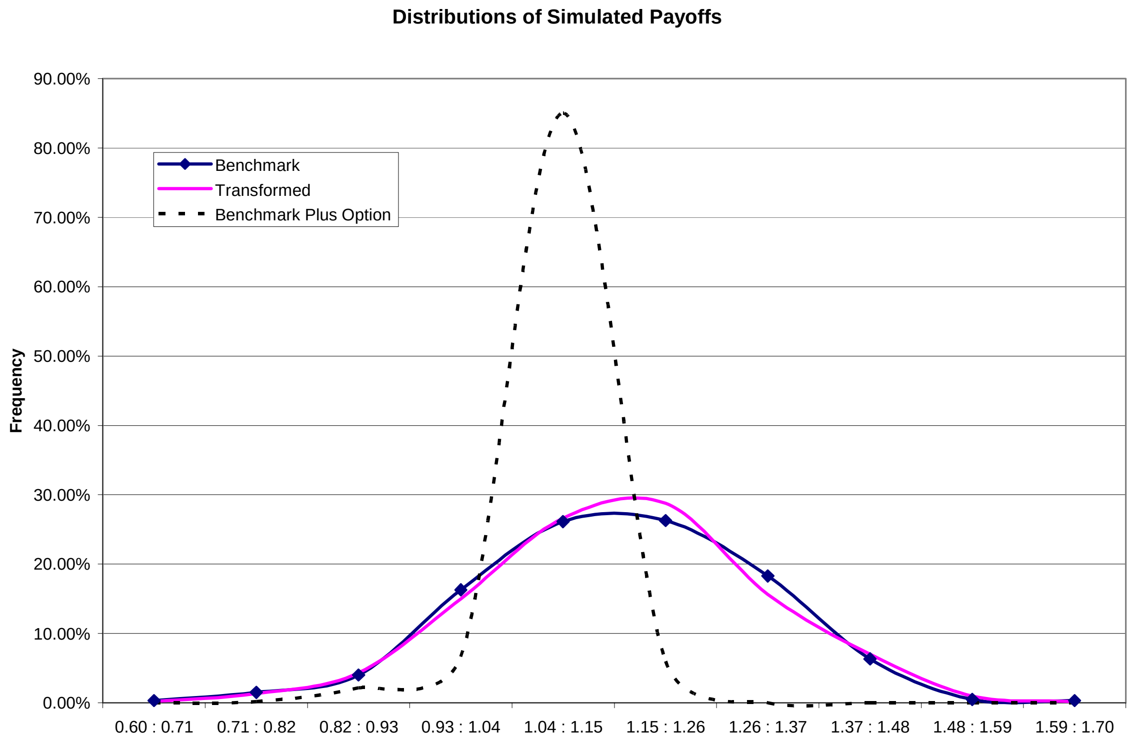


Figure 2

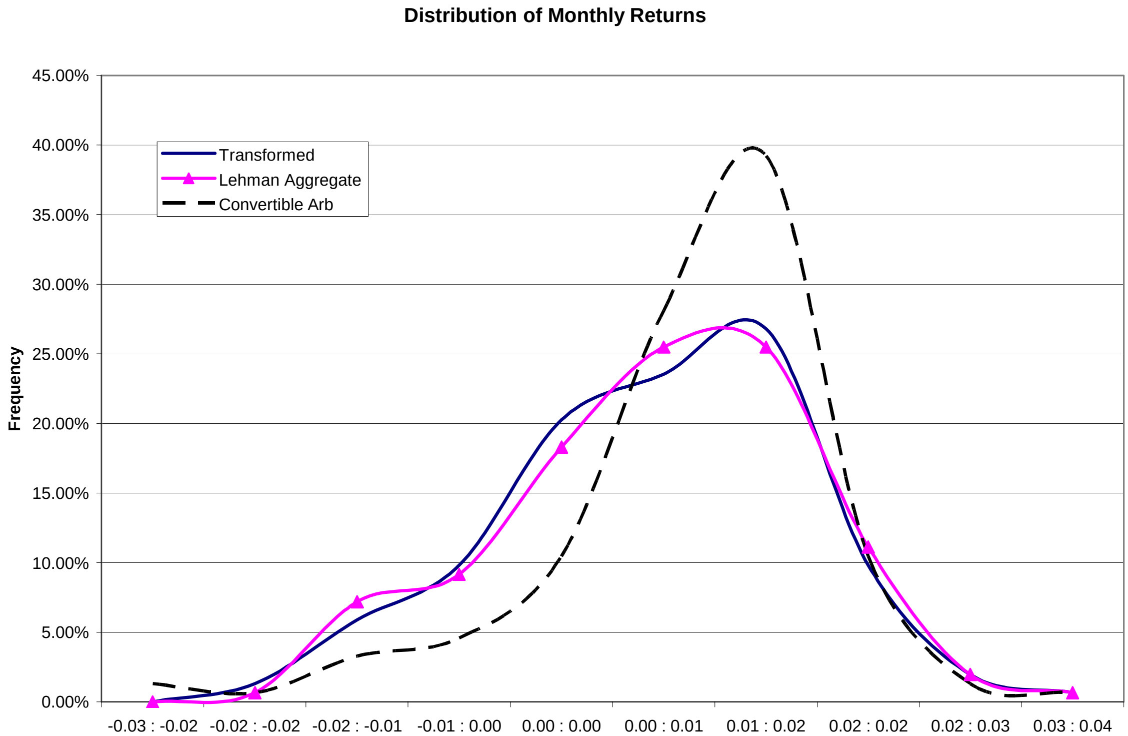
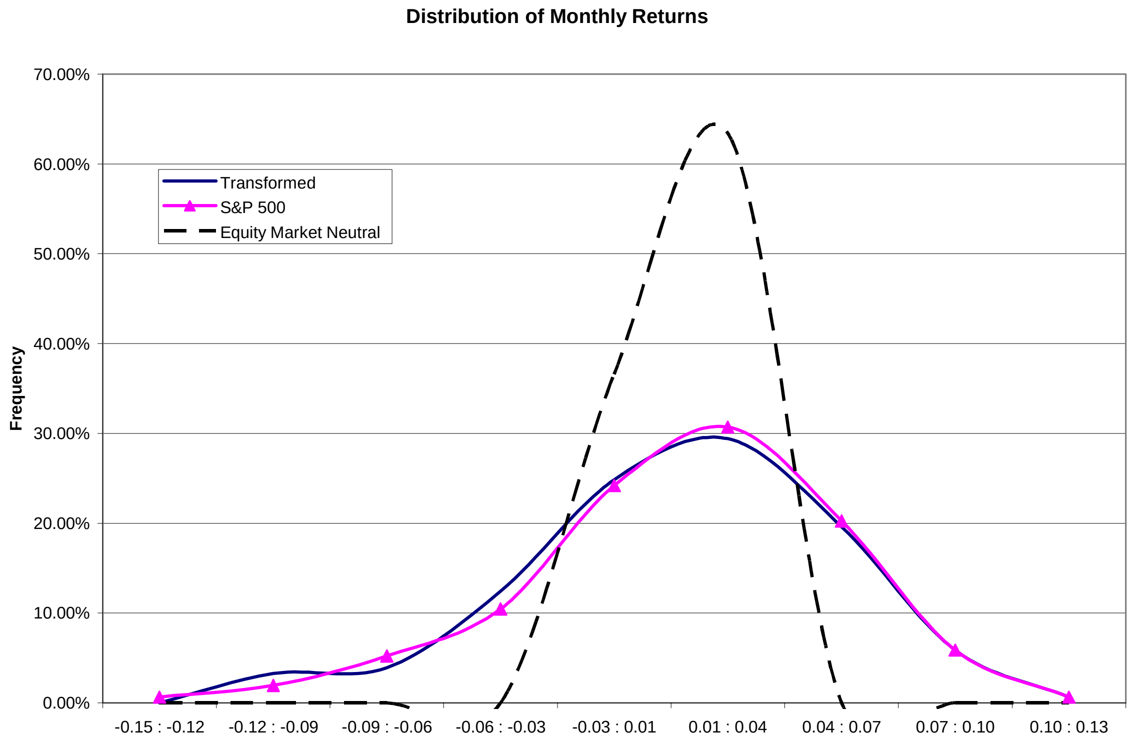

Distribution of Monthly Returns
45.00%
40.00%
35.00%
30.00%
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
Frequency
Transformed
Lehman Aggregate
Convertible Arb
-0.03 : -0.02
-0.02 : -0.02
-0.02 : -0.01
-0.01 : 0.00
0.00 : 0.00
0.00 : 0.01
0.01 : 0.02
0.02 : 0.02
0.02 : 0.03
0.03 : 0.04

Figure 3


Distribution of Monthly Returns
Frequency
70.00%
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
-0.15 : -0.12
-0.12 : -0.09
-0.09 : -0.06
-0.06 : -0.03
-0.03 : 0.01
0.01 : 0.04
0.04 : 0.07
0.07 : 0.10
0.10 : 0.13
Transformed
S&P 500
Equity Market Neutral

7. Tables

Table 1

	Annual						
	Mean	St Dev	Skewness	Kurtosis	Estimated Value of P	Sharpe Ratio	Adjusted Sharpe Ratio
Portfolio	9.14%	5.39%	-2.42	9.28	100.13%	0.79	0.67
Benchmark	15.01%	15.01%	0.09	-0.18	----	0.68	----
Transformed Return	15.00%	15.02%	0.17	-0.12	----	----	----

Table 2

	Monthly (January 1990-September 2002)					
	Mean	Std Dev	Skewness	Kurtosis	Sharpe Ratio	Benchmark
HFR Convertible Arbitrage	0.92%	0.98%	-1.343	3.210	0.525	Lehman Agg
HFR Distressed Securities	1.13%	1.84%	-0.623	5.362	0.392	Lehman Agg
HFR Fixed Income Arbitrage	0.71%	1.34%	-1.683	8.817	0.228	Lehman Agg
HFR Equity Hedge	1.45%	2.72%	0.134	1.098	0.382	S&P500
HFR Event Driven	1.14%	1.98%	-1.346	4.649	0.369	S&P500
HFR Merger Arbitrage	0.90%	1.31%	-2.778	11.383	0.375	S&P500
HFR Equity Market Neutral	0.85%	0.94%	-0.004	0.305	0.480	S&P500
HFR Emerging Markets Total	1.15%	4.59%	-0.729	3.286	0.163	S&P500
HFR FOF	0.84%	1.73%	-0.283	3.750	0.251	S&P500
HFR Macro	1.36%	2.54%	0.262	0.323	0.377	S&P500
CISDM Distressed securities	1.10%	2.04%	-0.824	3.989	0.341	Lehman Agg
CISDM Risk arbitrage	0.95%	1.27%	-1.305	6.453	0.430	S&P500
CISDM Global Emerging Markets	1.09%	4.66%	-1.056	9.138	0.147	S&P500
CISDM Global Macro	1.06%	1.99%	0.968	2.620	0.329	S&P500
CISDM Market Neutral	0.86%	0.43%	0.080	1.218	1.065	S&P500
CISDM Fund of Funds	0.78%	1.30%	-0.951	5.935	0.289	S&P500
CISDM Fund of Funds Diversified	0.81%	1.40%	-0.422	5.462	0.289	S&P500
CISDM Fund of Funds Niche	0.80%	1.46%	-0.094	4.475	0.269	S&P500
S&P 500	0.83%	4.35%	-0.484	0.559	0.098	
Lehman U.S. Aggregate	0.67%	1.09%	-0.280	0.097	0.243	

Table 3a

Monthly Estimates (January 1990-September 2002)	Mean	St Dev	Skewness	Kurtosis	Estimated Value of P	Sharpe Ratio	Adjusted Sharpe Ratio	Estimated Value of P*
Lehman U.S. Aggregate	0.67%	1.09%	-0.280	0.097	----	0.24	----	----
HFR Convertible Arbitrage Index	0.92%	0.98%	-1.343	3.210	99.64%	0.52	0.58	100.31%
Transformed Return	0.67%	1.09%	-0.264	0.048	----	----	----	----
HFR Distressed Securities Index	1.13%	1.84%	-0.623	5.362	99.79%	0.39	0.44	100.25%
Transformed Return	0.67%	1.08%	-0.252	0.411	----	----	----	----
HFR Fixed Income Arbitrage Index	0.71%	1.34%	-1.683	8.817	99.98%	0.23	0.26	100.05%
Transformed Return	0.67%	1.09%	-0.211	0.244	----	----	----	----
CISDM Distressed securities	1.10%	2.04%	-0.824	3.989	99.87%	0.34	0.37	100.14%
Transformed Return	0.67%	1.08%	-0.269	0.091	----	----	----	----

Table 3b

Monthly Estimates (January 1990-September 2002)	Mean	St Dev	Skewness	Kurtosis	Estimated Value of P	Sharpe Ratio	Adjusted Sharpe Ratio	Estimated Value of P*
S&P 500 Total Return Index	0.83%	4.35%	-0.48	0.56	----	0.10	----	----
HFR Equity Hedge Index	1.45%	2.72%	0.13	1.10	98.64%	0.38	0.41	100.76%
Transformed Return	0.83%	4.34%	-0.48	0.45	----	----	----	----
HFR Event Driven Index	1.14%	1.98%	-1.35	4.65	98.67%	0.37	0.40	100.55%
Transformed Return	0.83%	4.34%	-0.47	0.52	----	----	----	----
HFR Merger Arbitrage Index	0.90%	1.31%	-2.78	11.38	98.14%	0.38	0.53	100.32%
Transformed Return	0.83%	4.34%	-0.49	0.57	----	----	----	----
HFR Equity Market Neutral Index	0.85%	0.94%	0.00	0.31	98.35%	0.48	0.48	100.36%
Transformed Return	0.83%	4.33%	-0.45	0.41	----	----	----	----
HFR Emerging Markets Total Index	1.15%	4.59%	-0.73	3.29	99.75%	0.16	0.15	100.21%
Transformed Return	0.83%	4.34%	-0.48	0.53	----	----	----	----
HFR FOF Index	0.84%	1.73%	-0.28	3.75	99.47%	0.25	0.22	100.26%
Transformed Return	0.83%	4.34%	-0.50	0.69	----	----	----	----
HFR Macro Index	1.36%	2.54%	0.26	0.32	98.62%	0.38	0.41	100.62%
Transformed Return	0.83%	4.33%	-0.50	0.61	----	----	----	----
CISDM Risk arbitrage	0.95%	1.27%	-1.31	6.45	98.28%	0.43	0.50	100.43%
Transformed Return	0.83%	4.33%	-0.37	0.66	----	----	----	----
CISDM Global Emerging Markets	1.09%	4.66%	-1.06	9.14	99.64%	0.15	0.18	100.28%
Transformed Return	0.83%	4.33%	-0.43	1.01	----	----	----	----
CISDM Global Macro	1.06%	1.99%	0.97	2.62	98.73%	0.33	0.39	100.39%
Transformed Return	0.83%	4.33%	-0.44	0.72	----	----	----	----
CISDM Market Neutral	0.86%	0.43%	0.08	1.22	95.43%	1.06	1.15	100.42%
Transformed Return	0.83%	4.34%	-0.49	0.51	----	----	----	----
CISDM Fund of Funds	0.78%	1.30%	-0.95	5.94	99.10%	0.29	0.30	100.28%
Transformed Return	0.83%	4.34%	-0.47	0.73	----	----	----	----
CISDM Fund of Funds Diversified	0.81%	1.40%	-0.42	5.46	99.08%	0.29	0.31	100.29%
Transformed Return	0.83%	4.33%	-0.49	0.99	----	----	----	----
CISDM Fund of Funds Niche	0.80%	1.46%	-0.09	4.47	99.23%	0.27	0.27	100.24%
Transformed Return	0.83%	4.31%	-0.44	1.05	----	----	----	----

Table 4

Monthly (January 1995-September 2002)						
	Mean	**Std Dev**	**Skewness**	**Kurtosis**	**Sharpe Ratio**	**Benchmark**
Convertible 1	1.65%	1.19%	-0.520	2.618	1.041	Lehman Agg
Convertible 2	0.73%	0.52%	-0.485	1.082	0.622	Lehman Agg
Convertible 3	1.48%	1.29%	-1.529	9.009	0.836	Lehman Agg
Convertible 4	1.04%	1.01%	-0.450	0.986	0.626	Lehman Agg
Convertible 5	0.80%	0.63%	-0.249	0.414	0.634	Lehman Agg
Distress 1	1.00%	0.95%	-0.657	1.647	0.621	Lehman Agg
Distress 2	0.94%	1.02%	-0.693	3.196	0.521	Lehman Agg
Distress 3	0.69%	1.31%	-1.582	5.778	0.221	Lehman Agg
Distress 4	1.43%	1.27%	-0.573	2.724	0.811	Lehman Agg
Distress 5	1.65%	2.64%	-0.153	0.152	0.472	Lehman Agg
Event Driven 1	1.16%	0.90%	0.022	0.340	0.841	S&P500
Event Driven 2	0.89%	0.62%	-0.193	1.371	0.778	S&P500
Event Driven 3	0.48%	0.62%	-0.188	1.356	0.128	S&P500
Event Driven 4	0.86%	0.65%	-0.166	0.633	0.697	S&P500
Event Driven 5	0.95%	0.81%	-1.028	3.503	0.682	S&P500
Equity Hedge 1	1.00%	0.85%	0.363	0.379	0.699	S&P500
Equity Hedge 2	0.94%	0.76%	-0.287	7.016	0.703	S&P500
Equity Hedge 3	0.54%	1.48%	0.706	2.099	0.088	S&P500
Equity Hedge 4	1.30%	1.52%	-0.052	0.889	0.591	S&P500
Equity Hedge 5	1.91%	2.94%	-1.567	6.802	0.510	S&P500
Equity Market Neutral 1	0.72%	0.31%	1.008	3.705	1.018	S&P500
Equity Market Neutral 2	0.99%	0.62%	0.474	-0.619	0.940	S&P500
Equity Market Neutral 3	1.18%	0.76%	0.459	-0.605	1.025	S&P500
Equity Market Neutral 4	1.54%	2.34%	0.341	-0.416	0.484	S&P500
Equity Market Neutral 5	1.75%	3.78%	-0.114	2.603	0.356	S&P500
Merger Arbitrage 1	0.89%	0.75%	-0.881	4.678	0.652	S&P500
Merger Arbitrage 2	0.87%	0.79%	-1.109	4.415	0.585	S&P500
Merger Arbitrage 3	0.96%	0.92%	0.688	3.245	0.607	S&P500
Merger Arbitrage 4	0.77%	0.73%	-1.068	7.876	0.505	S&P500
Merger Arbitrage 5	1.12%	1.06%	-1.316	6.985	0.680	S&P500
S&P500	1.10%	4.61%	-0.655	0.383	0.151	
Lehman Aggregate	0.67%	1.05%	-0.067	0.294	0.258	

Table 5a

Monthly (January 1995-September 2002)								
	Mean	**St Dev**	**Skewness**	**Kurtosis**	**Estimated Value of P**	**Sharpe Ratio**	**Adjusted Sharpe Ratio**	**Estimated Value of P***
Lehman Aggregate	0.67%	1.05%	-0.07	0.29	----	0.26	----	----
Convertible 1	1.65%	1.19%	-0.52	2.62	99.16%	1.04	1.07	100.87%
Transformed Return	0.67%	1.03%	-0.06	0.64	----	----	----	----
Convertible 2	0.73%	0.52%	-0.49	1.08	99.61%	0.62	0.64	100.19%
Transformed Return	0.67%	1.04%	-0.12	0.05	----	----	----	----
Convertible 3	1.48%	1.29%	-1.53	9.01	99.24%	0.84	0.98	100.72%
Transformed Return	0.67%	1.04%	-0.13	0.20	----	----	----	----
Convertible 4	1.04%	1.01%	-0.45	0.99	99.59%	0.63	0.65	100.34%
Transformed Return	0.67%	1.04%	-0.12	0.12	----	----	----	----
Convertible 5	0.80%	0.63%	-0.25	0.41	99.63%	0.63	0.61	100.23%
Transformed Return	0.67%	1.04%	-0.14	0.09	----	----	----	----
Distress 1	1.00%	0.95%	-0.66	1.65	99.60%	0.62	0.64	100.37%
Transformed Return	0.67%	1.04%	-0.05	0.30	----	----	----	----
Distress 2	0.94%	1.02%	-0.69	3.20	99.71%	0.52	0.53	100.28%
Transformed Return	0.67%	1.04%	-0.07	0.20	----	----	----	----
Distress 3	0.69%	1.31%	-1.58	5.78	100.14%	0.22	0.12	99.95%
Transformed Return	0.67%	1.03%	-0.15	-0.14	----	----	----	----
Distress 4	1.43%	1.27%	-0.57	2.72	99.40%	0.81	0.83	100.69%
Transformed Return	0.67%	1.04%	-0.07	0.49	----	----	----	----
Distress 5	1.65%	2.64%	-0.15	0.15	99.78%	0.47	0.47	100.59%
Transformed Return	0.67%	1.04%	-0.17	0.09	----	----	----	----

Table 5b

Monthly (January 1995-September 2002)								
	Mean	**St Dev**	**Skewness**	**Kurtosis**	**Estimated Value of P**	**Sharpe Ratio**	**Adjusted Sharpe Ratio**	**Estimated Value of P**
S&P500	1.10%	4.61%	-0.66	0.38	----	0.15	----	----
Event Driven 1	1.16%	0.90%	0.02	0.34	96.65%	0.84	0.88	100.62%
Transformed Return	1.10%	4.59%	-0.65	0.34	----	----	----	----
Event Driven 2	0.89%	0.62%	-0.19	1.37	97.00%	0.78	0.80	100.40%
Transformed Return	1.10%	4.49%	-0.57	0.60	----	----	----	----
Event Driven 3	0.48%	0.62%	-0.19	1.36	100.11%	0.13	0.13	99.97%
Transformed Return	1.10%	4.49%	-0.57	0.61	----	----	----	----
Event Driven 4	0.86%	0.65%	-0.17	0.63	97.33%	0.70	0.73	100.32%
Transformed Return	1.10%	4.57%	-0.59	0.01	----	----	----	----
Event Driven 5	0.95%	0.81%	-1.03	3.50	97.19%	0.68	0.76	100.43%
Transformed Return	1.10%	4.57%	-0.54	0.40	----	----	----	----
Equity Hedge 1	1.00%	0.85%	0.36	0.38	96.67%	0.70	0.88	100.43%
Transformed Return	1.10%	4.60%	-0.60	0.28	----	----	----	----
Equity Hedge 2	0.94%	0.76%	-0.29	7.02	96.78%	0.70	0.85	100.39%
Transformed Return	1.10%	4.52%	-0.67	1.27	----	----	----	----
Equity Hedge 3	0.54%	1.48%	0.71	2.10	100.27%	0.09	0.09	99.88%
Transformed Return	1.10%	4.58%	-0.69	0.28	----	----	----	----
Equity Hedge 4	1.30%	1.52%	-0.05	0.89	97.79%	0.59	0.63	100.66%
Transformed Return	1.10%	4.57%	-0.61	0.11	----	----	----	----
Equity Hedge 5	1.91%	2.94%	-1.57	6.80	98.18%	0.51	0.55	101.01%
Transformed Return	1.10%	4.60%	-0.65	0.43	----	----	----	----
Equity Market Neutral 1	0.72%	0.31%	1.01	3.70	95.51%	1.02	1.13	100.25%
Transformed Return	1.10%	4.51%	-0.53	-0.18	----	----	----	----
Equity Market Neutral 2	0.99%	0.62%	0.47	-0.62	93.05%	0.94	1.66	100.49%
Transformed Return	1.10%	4.60%	-0.65	0.37	----	----	----	----
Equity Market Neutral 3	1.18%	0.76%	0.46	-0.60	91.16%	1.02	2.08	100.64%
Transformed Return	1.10%	4.58%	-0.57	-0.17	----	----	----	----
Equity Market Neutral 4	1.54%	2.34%	0.34	-0.42	98.20%	0.48	0.54	100.69%
Transformed Return	1.10%	4.59%	-0.64	0.15	----	----	----	----
Equity Market Neutral 5	1.75%	3.78%	-0.11	2.60	98.88%	0.36	0.40	100.82%
Transformed Return	1.10%	4.58%	-0.66	0.62	----	----	----	----
Merger Arbitrage 1	0.89%	0.75%	-0.88	4.68	97.31%	0.65	0.74	100.39%
Transformed Return	1.10%	4.57%	-0.62	0.61	----	----	----	----
Merger Arbitrage 2	0.87%	0.79%	-1.11	4.41	97.69%	0.58	0.66	100.33%
Transformed Return	1.10%	4.59%	-0.57	0.36	----	----	----	----
Merger Arbitrage 3	0.96%	0.92%	0.69	3.24	97.18%	0.61	0.77	100.39%
Transformed Return	1.10%	4.58%	-0.63	0.64	----	----	----	----
Merger Arbitrage 4	0.77%	0.73%	-1.07	7.88	97.75%	0.50	0.64	100.23%
Transformed Return	1.10%	4.57%	-0.67	0.90	----	----	----	----
Merger Arbitrage 5	1.12%	1.06%	-1.32	6.99	96.76%	0.68	0.86	100.54%
Transformed Return	1.10%	4.56%	-0.52	0.76	----	----	----	----

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